Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 5: Algebra III

5.1 Learning Intentions

After this week's lesson you will be able to;

- Solve quadratic equations
- Interpret a graph to form a polynomial and vice versa
- Solve simultaneous equations (linear and non-linear)

5.2 Specification

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.2 Solving equations	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x) = g(x)$, with $f(x) = ax+b$, $g(x) = cx+d$ where $a, b, c, d \in \mathbf{Q}$ • $f(x) = g(x)$ with $f(x) = \frac{a}{bx+c} \pm \frac{p}{qx+r}$; $g(x) = \frac{e}{f}$ where $a, b, c, e, f, p, q, r \in \mathbf{Z}$ • $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable) where $a, b, c \in \mathbf{Q}$ and interpret the results – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is $\pm$ 1 in the linear equation) and interpret the results – form quadratic equations given whole number roots	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$ with $f(x) = \frac{ax+b}{ex+f} \pm \frac{cx+d}{qx+r}$; $g(x) = k$ where $a, b, c, d, e, f, q, r \in \mathbf{Z}$ – use the Factor Theorem for polynomials – select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to • cubic equations with at least one integer root • simultaneous linear equations with three unknowns • one linear equation and one equation of order 2 with two unknowns and interpret the results

5.3 Chief Examiner's Report

Student should also be encouraged to construct algebraic expressions or equations to model these situations, and / or to draw diagrams to represent them.

5.4 Quadratic Equations

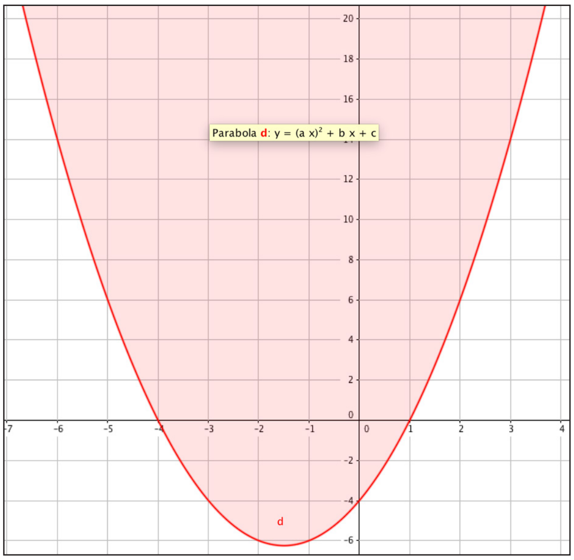
Quadratic equations are defined as equations that have degree 2. Typically, they have an x squared term an x term and a constant. However only the x squared term is necessary to be regarded as a quadratic equation.

With degree two, this means that a quadratic equation is should have two solutions. These solutions can be distinct or the same but there will still be two solutions even if they are the same in value.

The general form of these equations is

$$ax^2 + bx + c = 0$$

Quadratic equations are often better understood through the medium of graphs.



The best way to get an understanding of these graphs is to generate them yourself on a computer and alter the coefficients a, b and c and view the effect this has on the graph. To do this use the software GeoGebra (found at www.geogebra.org).

You should see that:

- a affects the width of the u shape graph (if a is negative, the graph is inverted to be more of a n-shape)
- b pivots the graph about the y-intercept
- c controls the y-intercept (shifts the graph up and down).

In solving quadratic equations, we have three main approaches:

1) Using a graph
2) Guide Number Method
3) Quadratic Formula

Using a graph:

In using a graph, we are identifying the points where the function intersects or crosses the x-axis. In most cases there will be two of these points. Below is a different example to that in the video.

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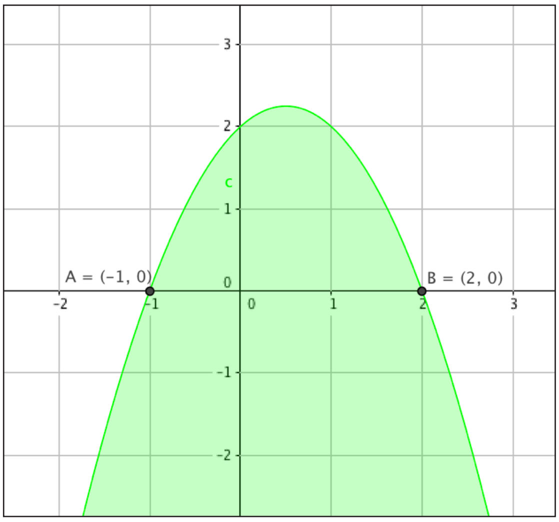


As we can see with this function it is an n shape so that means our function here is a negative x squared. Our quadratic is:

$$-x^2 + x + 2 = 0$$

From our graph we can see that the two solutions to this equation are:

$$x = -1 \text{ or } x = 2$$

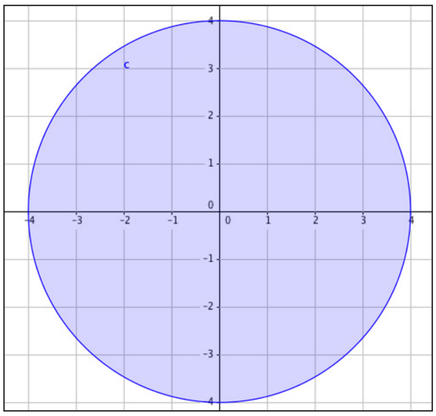
<u>Using the guide number method:</u>

Solve $3x^2 + 5x - 12 = 0$

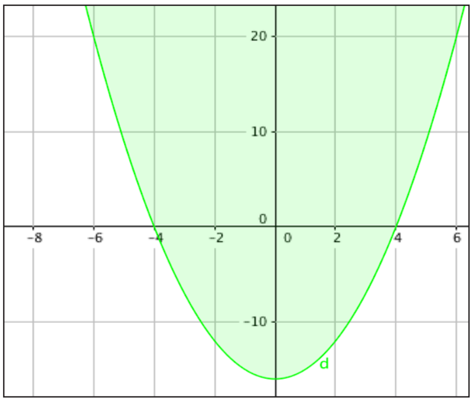
$3x^2 + 5x - 12 = 0$	Multiply x squared coefficient and constant (3 x -12 = -36)
$3x^2 - 4x + 9x - 12 = 0$	Use factors of -36 that sum to give +5
$x(3x - 4) + 3(3x - 4) = 0$	Factorise through H.C.F.
$(x + 3)(3x - 4) = 0$	Distributive property to create factors
$x + 3 = 0 \text{ or } 3x - 4 = 0$	If two terms mult. To give 0 then one at least must be zero.
$x = -3 \text{ or } 3x = 4$	
$x = -3 \text{ or } x = {}^4/_3$	

5.5 Simultaneous Equations

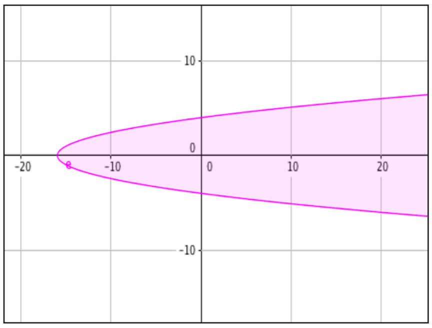
Having looked at some of these equations in the previous week we will now look at a linear equation with a non-linear equation.
Below are the examples of non-linear equations:



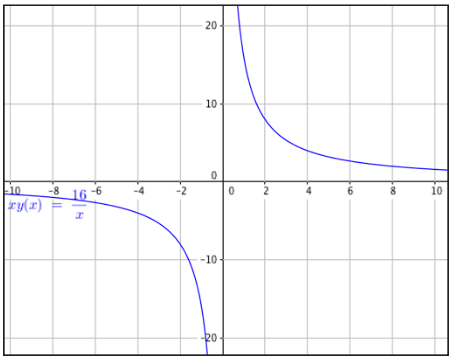
$x^2 + y^2 = 16$



$x^2 = 16$

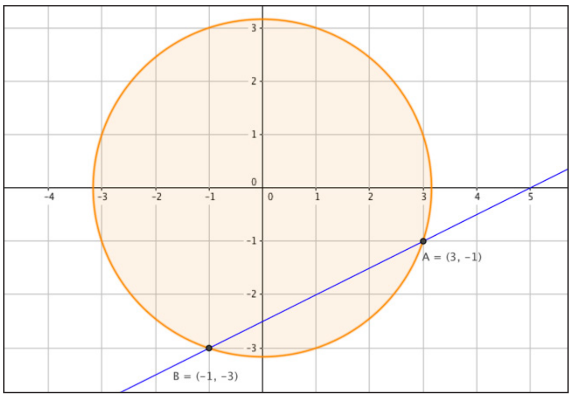


$y^2 = 16$



$xy = 16$

When we a solving a linear equation with one of the above types of non-linear equation we are finding the coordinates of the points of intersection between the linear (line) and the non-linear (curve).



In solving these equations together, here are the steps we follow:

1) Identify the linear and non linear equations

$$x^2 + y^2 = 10 \text{ Non- Linear}$$
$$x - 2y = 5 \text{ Linear}$$

2) Rearrange the linear equation to have x = or y =

$$x - 2y = 5$$
$$x = 5 + 2y$$

3) Sub this expression into the non-Linear equation and solve

$$x^2 + y^2 = 10$$

$$(5 + 2y)^2 + y^2 = 10$$

$$25 + 20y + 4y^2 + y^2 = 10$$

$$15 + 20y + 5y^2 = 0$$

$$3 + 4y + y^2 = 0$$

$$y = -1 \; or \; y = -3$$

4) Sub each y value into linear expression from step 2 to get x coordinates.

$$x = 3 \; or \; x = -1$$

Therefore, the two points of intersection are:

(3, -1) and (-1, -3).

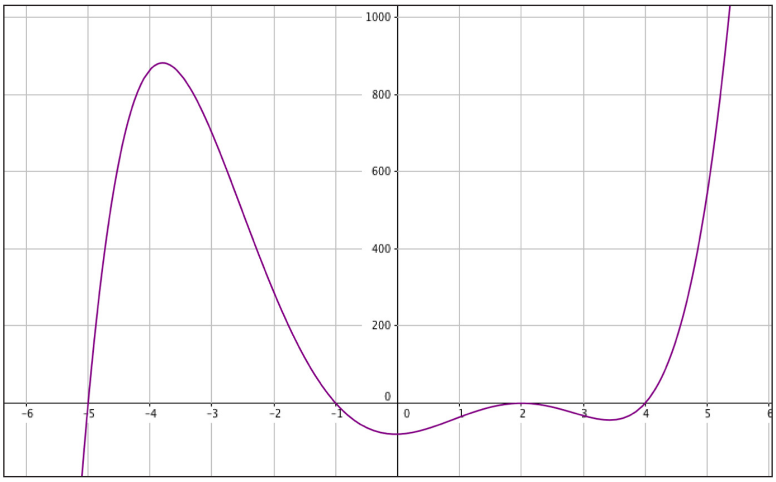
Which we can verify from the graph of both of these equations in the video.

5.6 Estimating Polynomials

For this we need to explain the factor theorem. With a polynomial, if it has a root that is x = a then the corresponding factor is (x – a). We can use this idea to estimate a polynomial.

If a graph crosses the x-axis at x = a then x – a is a factor. However, if a graph appears to "bounce off" the x-axis at x = a. This means that that root has multiplicity 2, i.e. we have the same root twice.

Following the video, establish the polynomial of the below graph:



We can also use this idea to sketch a graph of a polynomial, once we have it in factor form such as the below polynomial:

$$y = x^2(x-5)(x+2)(x+4)$$

Using the polynomial we can establish the roots:

Roots =

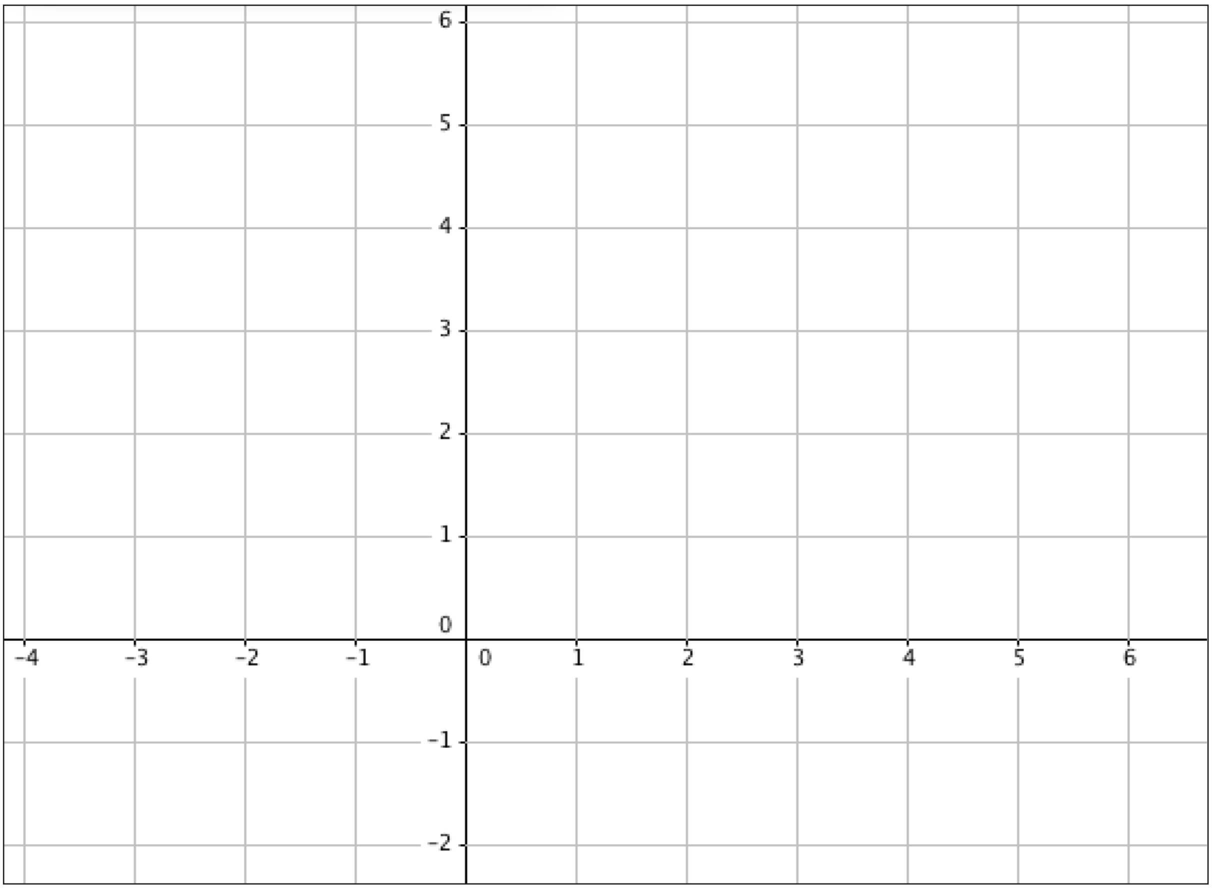
Remember:

Odd degree = Arms face opposite directions

Even degree = Arms face same direction

Leading Coefficient (coefficient with highest power of x) if negative, right arm points down and if positive right arm points upwards.

On the below grid, sketch the polynomial from the video:



You may be asked to sketch a polynomial when not given the roots or factors but through long division and trial and error you can establish them.

5.7 Recap of the Learning Intentions

After this week's lesson you will be able to;

- -Solve quadratic equations
- -Interpret a graph to form a polynomial and vice versa
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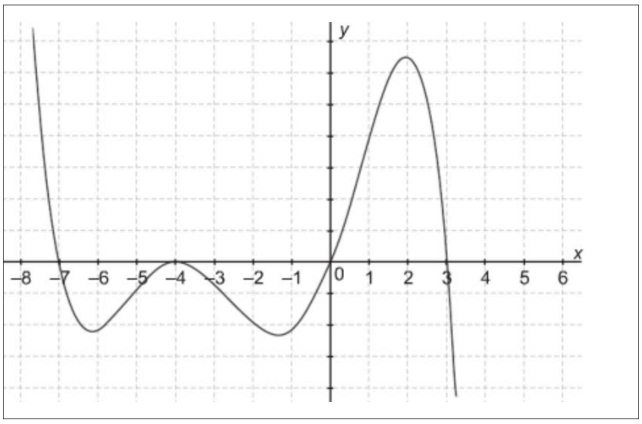
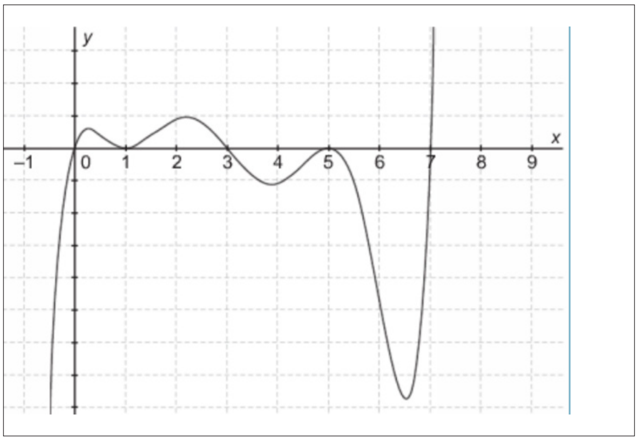
5.8 Homework Task

Find the roots and thus the polynomial of the below graphs





Roots:

Factors:

y =

Roots:

Factors:

y =

Solve the following simultaneous equation:

$$x^2 + y^2 - 13 = 0$$
$$5x - y = -13$$

5.9 Solutions to 4.9

$x + y + z = 16$
$\frac{5}{2}x + y + 10z = 40$
$2x + \frac{1}{2}y + 4z = 21$

Firstly, remove the denominators by multiplying across the required equation

$x + y + z = 16$
$\frac{(2)5}{2}x + (2)y + (2)10z = (2)40$
$(2)2x + \frac{(2)1}{2}y + (2)4z = (2)21$

$x + y + z = 16$ 1
$5x + 2y + 20z = 80$ 2
$4x + y + 8z = 42$ 3

Now we chose a pair of equations and eliminate one variable. 1 and 3 to eliminate y

$x + y + z = 16$ 1
$4x + y + 8z = 42$ 3
1 – 3 will eliminate the y
$-3x - 7z = -26$ 4 (new equation)

Now repeat with a different pair (1 and 2) to eliminate y (must be the same variable eliminated)

$x + y + z = 16$ 1
$5x + 2y + 20z = 80$ 2

Multiply number 1 by – 2 to give us -2y on the top. So we can add 1 and 2 to eliminate the y's.

$-2x - 2y - 2z = -32$ 1
$5x + 2y + 20z = 80$ 2

Now we add 1 and 2 to give us another new equation (5)

$3x + 18z = 48$ 5 (new equation)

Now we use 4 and 5 to get a value for x or z

$-3x - 7z = -26$4
$3x + 18z = 48$ 5

We can add 4 and 5 to eliminate the x's.

$11z = -22$
$\boldsymbol{z = 2}$

Sub 10 in for z in equation 4 to get a value for x

$-3x - 7(2) = -26$
$\boldsymbol{x = 4}$

Now sub 4 in for x and 2 in for z in one of the first 3 equations to get a value for y

$x + y + z = 16$ 1

$4 + y + 2 = 16$ 1

$\mathbf{y = 10}$

FINAL ANSWER: $\boldsymbol{x = 4, y = 10\ and\ z = 2}$